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BMATE201

Second Semester B.E./B.Tech. Degree Examination, June/July 2024 Mathematics – II for EEE Stream

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
 2. VTU Formula Hand Book is permitted.
 3. M : Marks , L: Bloom's level , C: Course outcomes.*

Module – 1			M	L	C
Q.1	a.	Find the directional derivatives of $\phi = x^2yz + 4xz^2$, at $(1, 2, -1)$ along $2\hat{i} - \hat{j} - 2\hat{k}$.	7	L3	CO1
	b.	Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$, where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.	7	L3	CO1
	c.	Show that the vector, $\vec{F} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational.	6	L2	CO1
OR					
Q.2	a.	Find the work done in moving a particle in the Force field $F = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the straight line from $(0, 0, 0)$ to $(2, 1, 3)$.	7	L3	CO1
	b.	Using Green's theorem, evaluate $\oint (xy + y^2)dx + x^2dy$ over the region bounded by the curves $y = x$ and $y = x^2$.	7	L3	CO1
	c.	Using modern mathematical tools, write a code to find the divergence and curl of the vector $x^2yz\hat{i} + y^2zx\hat{j} + z^2xy\hat{k}$.	6	L2	CO5
Module – 2					
Q.3	a.	Define a subspace. Show that the intersection of two subspaces of a vector space V is also a subspace of V .	7	L2	CO2
	b.	Define a basis for a vector space. Determine whether or not the vectors : $(2, 2, 1), (1, 3, 7), (1, 2, 2)$ form a basis of R^3 .	7	L2	CO2
	c.	Show that $T : R^2 \rightarrow R^3$ given by $T(x, y) = (x + y, x - y, y)$ is a linear transformation.	6	L2	CO2
OR					
Q.4	a.	Define linearly independent set of vectors and linearly dependent set of vectors. Show that the vectors $(1, 4, 9), (3, 1, 4), (9, 3, 12)$ are linearly dependent.	7	L2	CO2
	b.	Verify the Rank-Nullity theorem for $T : R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.	7	L2	CO2
	c.	Using the modern mathematical tool, write the code to represent the reflection transformation $T : R^2 \rightarrow R^2$ and to find the image of vector $(10, 0)$ when it is reflected about the y -axis.	6	L2	CO5
Module – 3					
Q.5	a.	Find the Laplace Transform of, (i) $e^{-3t}(2\cos 5t - 3\sin 5t)$ (ii) $\frac{\cos at - \cos bt}{t}$	7	L2	CO3

	b.	Find the Laplace transform of the triangular wave function, $f(t) = \begin{cases} t, & \text{if } 0 \leq t \leq a \\ 2a - t, & \text{if } a \leq t \leq 2a \end{cases}$	7	L2	CO3										
	c.	Express $f(t) = \begin{cases} t^2, & 1 < t \leq 2 \\ 4t, & t > 2 \end{cases}$ in terms of Heaviside unit step function and hence find $L(f(t))$.	6	L3	CO3										
OR															
Q.6	a.	Find $L^{-1} \left[\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6} \right]$.	7	L2	CO3										
	b.	Find $L^{-1} \left[\frac{1}{s(s^2 + a^2)} \right]$ using convolution theorem.	7	L2	CO3										
	c.	Solve the differential equation by using Laplace Transform method. $y'' + 6y' + 9y = 12t^2 e^{-3t}$, $y(0) = y'(0) = 0$	6	L3	CO3										
Module – 4															
Q.7	a.	By Newton-Raphson method, find the root of $x \sin x + \cos x = 0$, near $x = \pi$. Carry out the iteration up to four decimal places of accuracy.	7	L2	CO4										
	b.	Using Lagrange's interpolation formula, find y at $x = 2$, using the points $(0, -12), (1, 0), (3, 6), (4, 12)$	7	L2	CO4										
	c.	Using Simpson's $\left(\frac{1}{3}\right)^{\text{rd}}$ rule, evaluate $\int_0^{0.6} e^{-x^2} dx$ by taking seven ordinates.	6	L3	CO4										
OR															
Q.8	a.	Find a real root of the equation $x^3 - 4x - 9 = 0$ correct to three decimal places by the method of False position in $(2, 3)$	7	L2	CO4										
	b.	Construct Newton's forward interpolation polynomial for the data : <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>f(x)</td> <td>1</td> <td>2</td> <td>1</td> <td>10</td> </tr> </table>	x	0	1	2	3	f(x)	1	2	1	10	7	L2	CO4
x	0	1	2	3											
f(x)	1	2	1	10											
	c.	Evaluate $\int_0^1 \frac{dx}{(1+x)^2}$ by using Simpson's $\left(\frac{3}{8}\right)^{\text{th}}$ rule, by taking 6 equal intervals.	6	L3	CO4										
Module – 5															
Q.9	a.	Use Taylor series method to find $y(0.2)$ from $\frac{dy}{dx} = 2y + 3e^x$, with $y(0) = 0$.	7	L3	CO5										
	b.	Using R-K method of order 4, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$.	7	L3	CO5										
	c.	Applying Milne's Predictor-Corrector method, find $y(0.4)$, from $\frac{dy}{dx} = 2e^x - y$, given that, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$, $y(0.3) = 2.090$	6	L3	CO5										
OR															

Q.10	a. Solve by using modified Euler's method, $y' = 1 + \frac{y}{x}$, $y(1) = 2$ at $x = 1.2$ and $x = 1.4$.	7	L3	CO5
	b. Using the Runge-Kutta method of fourth order find $y(1.1)$, given $\frac{dy}{dx} = xy^{\frac{1}{3}}$, taking $h = 0.1$, $y(1) = 1$.	7	L3	CO5
	c. Using modern mathematical tools, write a code to find $y(1.4)$, given $\frac{dy}{dx} = x^2 + \frac{y}{2}$, $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$, by Milne's Predictor and Corrector method.	6	L3	CO5
